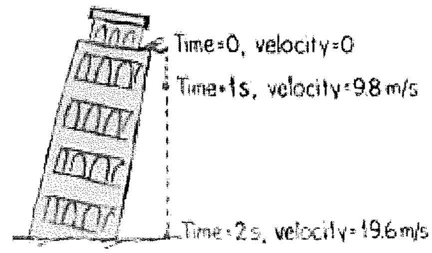
Galileo and the Inclined Plane

Introduction

In the early seventeenth century, Galileo makes a claim (or hypothesis) concerning the natural motion ("natural motion" is Aristotelian language) of a freely falling body. Galileo claimed that a naturally falling object will gain equal amounts of velocity in equal amounts of time. If correct, this means that i) the object's speed increases as it falls and ii) the rate at which it picks up speed does not change during the fall. The cartoon at the right illustrates the idea.



Galileo's hypothesis: A falling object picks up the same amount of speed in the second (and third and fourth) second of travel as it did during the first second until it hits the ground. Notice that the object travels further in the 2nd second of motion than during the first.

Picture from Hewitt et al (1994), *Conceptual Physical Science* (New York: HarperCollins).

However, practical problems make it difficult for Galileo to put such a claim to any experimental test. Freely falling objects move too quickly to study or record the motion directly. Because seventeenth century clocks could not record the short times involved, Galileo tried to slow down the motion by replacing the falling object with a ball rolling down a gently inclined plane.

> *"In order to make use of motions as slow as possible ... I also thought of making moveables descend along an inclined plane not much raised above the horizontal" (Galileo, Two New Sciences, p. 87).*

Notice that Galileo guessed (or assumed) that objects descending an incline speed up in exactly the same way falling objects do! Essentially, he reasoned as follows. A ball rolling down a steep incline will pick up speed faster than a ball rolling down a gentle incline, but the way in which its speed increases will be the same. Freefall, he reasoned, is simply equivalent to a vertical ramp. The character of the ball's motion in freefall should be the same as the character of the motion of a ball "falling" down the inclined plane.

Another technological problem arose in measuring the velocity. If Galileo's hypothesis is right, the velocity of the freely falling object changes continuously. Galileo cannot measure changing velocities directly, but he can measure distances and times. Galileo, therefore, uses math to transform his claim about **times and velocities** into a claim about **times and distances**. His mathematical argument is summarized below:

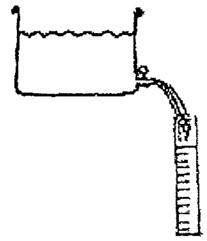
If an object gains speed at a steady rate and
if the object is released from rest,
then the total distance traveled by the object will be proportional to the square of the time needed for that travel.

For example, if an object released from rest travels, say for three seconds, it will travel 3*3 or 9 times as far as it would if it traveled for only one second after being released from rest. (Can you figure out the logic behind Galileo's mathematical argument? How can he derive the final statement from the first two?)

In this lab, you will put yourself in Galileo's shoes. Your job is to collect and present convincing evidence in support of Galileo's idea about the motion of falling bodies. You will face some of the experimental design questions and try to replicate the results of his famous experiment using an experimental setup similar to the one Galileo describes in his *Two New Sciences* of 1638.

You will also use a modern device to record the travel of a freely falling object. This will enable you to test whether the character of freefall motion is the same as that for the ball rolling down the ramp.

How do you measure time with 17th century clocks?



Stopwatches as we know them did not exist in Galileo's time. It was not until the 1700s that craftsmen began to make small reliable mechanical clocks. In his book *Two New Sciences*, Galileo describes the clock he uses:

> *For the measurement of time, we employed a large vessel of water placed in an elevated position; to the bottom of this vessel was soldered a pipe of small diameter giving a thin jet of water, which we collected in a small glass during the time of each descent... the water thus collected was weighed, after each descent, on a very accurate balance; the difference and ratios of these weights gave us the differences and ratios of the times...*

The basic principle behind this clock seems sound, but what guarantee is there that this "clock" doesn't gain or lose time? Since you want to make a specific mathematical claim involving time measurements, it is critical that your time measurements be accurate.

<u>Prelab Questions:</u>

- In this passage, Galileo seems to be assuming that there is a specific mathematical relationship between the amount of water collected and the amount of elapsed time. What assumption does Galileo seem to be making about the mathematical relationship between the amount of water and the amount of time?
- Would you expect this clock to run steadily? Why or why not? (Hint: Suppose the tank were nearly full and you let it run for 1 sec and collected the water; then you did the same thing, this time starting with the water level much lower than before. Would you collect the same amount of water in both cases? What might you expect?)
- Optional Challenge: Explain a procedure that you can perform to test whether this clock keeps good time. Remember that you have no 20th century clocks to compare against!

Appendix: Galileo's "Lab Report"

Galileo reports the inclined plane experiment in his book "Two New Sciences." The book is written as a series of intellectual conversations among three characters (Simplicio, Sagredo and Salviati) concerning the work of "the author" (Galileo). In this excerpt, Simplicio, Sagredo and Salviati discuss the inclined plane experiment. (Note: One bracchio is about 0.6 meters. Bracchio is the Italian word for arm.)

> "**Simplicio**: Really I have taken more pleasure from this simple and clear reasoning of Sagredo's than from the (for me) more obscure demonstration of the Author, so that I am better able to see why the matter must proceed in this way, once the definition of uniformly accelerated motion has been postulated and accepted. But I am still doubtful whether this is the acceleration employed by nature in the motion of her falling heavy bodies. Hence, for my understanding and for that of other people like me, I think that it would be suitable at this place [for you] to adduce some experiment from those (of which you have said that there are many) that agree in various cases with the demonstrated conclusions.
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> **Salviati**: Like a true scientist, you make a very reasonable demand, for this is usual and necessary in those sciences which apply mathematical demonstrations to physical conclusions, as may be seen among writers on optics, astronomers, mechanics, musicians, and others who confirm their principles with sensory experience that are the foundations of all the resulting structure. I do not want to have it appear a waste of time on our part, [as] if we had reasoned at excessive length about this first and chief foundation upon which rests an immense framework of infinitely many conclusions--of which we have only a tiny part put down in this book by the Author, who will have gone far to open the entrance and portal that has until now been closed to speculative minds. Therefore as to the experiments: the Author has not failed to make them, and in order to be assured that the acceleration of heavy bodies falling naturally does follow the ratio expounded above, I have often made the test in the following manner, and in his company.
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> In a wooden beam or rafter about twelve braccia long, half a braccio wide, and three inches thick, a channel was rabbeted in along the narrowest dimension, a little over an inch wide and made very straight; so that this would be clean and smooth, there was glued within it a piece of vellum, as much smoothed and cleaned as possible. In this there was made to descend a very hard bronze ball, well rounded and polished, the beam having been tilted by elevating one end of it above the horizontal plane from one to two braccia, at will. As I said, the ball was allowed to descend along the said groove, and we noted (in the manner I shall presently tell you) the time that it consumed in running all the way, repeating the same process many times, in order to be quite sure as to the amount of time, in which we never found a difference of even the tenth part of a pulse-beat.
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> This operation being precisely established, we made the same ball descend only one-quarter the length of this channel, and the time of its descent being measured, this was found to be precisely one-half the other. Next making the experiment for other lengths, examining now the time for the whole length [in comparison] with the time of one-half, or with that of two-thirds, or of three-quarters, and finally with any other division, by experiments repeated a full hundred times, **the spaces were always found to be to one another as the squares of the times**. And this [held] for all inclinations of the plane: that is, of the channel in which the ball was made to descend, where we observed also that the times of descent for diverse inclinations maintained among themselves accurately that ratio that we shall find later assigned and demonstrated by our Author.
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> As to the measure of time, we had a large pail filled with water and fastened from above, which had a slender tube affixed to its bottom, through which a narrow thread of water ran; this was received in a little beaker during the entire time that the ball descended along the channel or parts of it. The little amounts of water collected in this way were weighed from time to time on a delicate balance, the differences and ratios of the weights giving us the differences and ratios of the times, and with such precision that, as I have said, these operations repeated time and time again never differed by any notable amount.
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> **Simplicio**: It would have given me great satisfaction to have been present at these experiments. But being certain of your diligence in making them and your fidelity in relating them, I am content to assume them as most certain and true."

The excerpt is taken from Galileo, *Two new sciences*, tranl. Stillman Drake (Madison: University of Wisconsin Press, 1974), pp. 169-70.